Unsteady Heat Transfer Problems Related to a High-Power Laser Flow Loop

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Introduction

A PULSED closed-cycle electric discharge laser (EDL) is a very important device for achieving high energy intensity laser beams. Figure 1 shows a simplified sketch of the EDL. A working gas flows through a closed loop as in a closed circuit wind tunnel used in aerodynamic applications. The flow is circulated by the fan.[1]

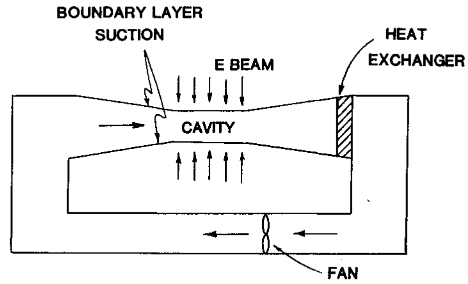

Fig. 1 Flow loop of a pulsed closed cycle electric discharge laser.

In the cavity section, pulsed electron beam energy is deposited into the flowing gas to achieve population inversion leading to lasing action. Pulse frequencies of 100 Hz are typical, and each pulse typically lasts for 5 μs. As far as the fluid dynamics of the cavity flow is concerned, the energy deposition is instantaneous, and, therefore, the process can be described as an instantaneous constant volume heating process. Thus, the gas pressure and temperature are instantaneously increased by the energy deposition in the cavity. Because of this, shock waves traveling away from the cavity in both upstream and downstream directions are created. Acoustic mufflers for attenuating these shocks are located upstream and downstream of the cavity. The pressure in the cavity region relaxes back to near its original value in a short period of time.[2]

However, the temperature discontinuities created by the energy deposition do not relax in a short period of time. A hot region of gas remains long after the pressure waves have traveled away. This hot slug of gas flows from the cavity at the speed of the flow and encounters the heat exchanger located downstream. The heat exchanger not only removes the heat from the hot slug of gas but also causes longitudinal mixing of this hot slug with the previous cold slug which passed through the heat exchanger. The fan also causes mixing and, thus, the working gas enters the cavity at a lower temperature. For the cavity medium to produce a high-quality output laser beam, there are stringent requirements on density homogeneity. Typically $(\Delta\rho/\rho)$ rms $\sim 10^{-4}$. Variations in density are due to several sources. One source is the unsteady temperature profile at the inlet of the heat exchanger, shown in Fig. 2. The pulse width in Fig. 2 is small compared to the interpulse time, and, therefore, the pulse width is considered infinitesimal.

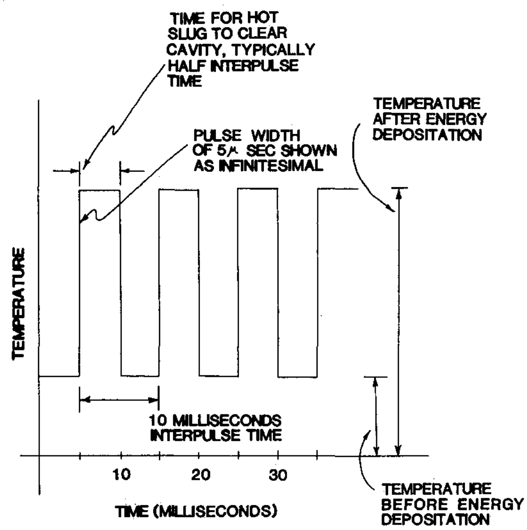

Fig. 2 Variation of the inlet temperature to the heat exchanger with time.

A second source of density variation is the temperature balance between the gas and the solid surfaces in the loop the gas temperature changes with time. Estimating the relaxation times and density changes requires a knowledge of unsteady heat transfer. One unique feature of these unsteady heat transfer problems is that the fluid bulk temperature fluctuates as opposed to problems where the body surface temperature fluctuates. Available literature shows that even though a large number of papers are published concerning unsteady heat transfer, little work is available which considers fluid bulk temperature variation, when a time dependent source term in the energy equation is involved.

The generic problem discussed is a general three-dimensional compressible turbulent flow past a body whose surface temperature fluctuates and in which the free-stream temperature also fluctuates. We propose starting with simplified idealized models and continue further work with increasing degrees of complexity.

The present paper deals with the incompressible flow past a flat plate. The wall temperature is assumed to be constant at all times. Initially the fluid is at the same temperature as the wall. At time $t = 0$ the bulk temperature of the fluid increases as a step function from T_w to T_∞. It is proposed to solve for the thermal boundary layer and wall heat transfer as a function of time using the momentum integral method.

Mathematical Formulation

Assuming two-dimensional incompressible flow with negligible viscous dissipation and constant flow properties, the equation for the temperature distribution T in the boundary layer is given by

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{1}$$

u and v are the velocity components in the boundary layer and are given by the well known Blasius relations. The thermal diffusivity of the gas is α. The boundary and initial conditions on the temperature are

$$T = T_w, \text{ at } t = 0 \tag{2}$$

$$T = T_w, \text{ at } y = 0 \text{ and } T = T_\infty, \text{ as } y \to \delta_T, \text{ for } t > 0 \tag{3}$$

where δ_T is the thickness of the thermal boundary layer.

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An integral approach similar to Goodman[3] is used to solve Eqs. (1) to (3). The velocity profile u is assumed to be of the form

$$u = u_\infty\left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right] \tag{4}$$

u_∞ is the constant free-stream velocity and δ is the velocity boundary layer thickness and is given by the relation

$$\delta(x) = 4.64\left[\frac{\nu x}{u_\infty}\right]^{1/2} \tag{5}$$

where ν represents the kinematic viscosity of the gas. For the temperature distribution in the thermal boundary layer, a profile of the form

$$T_\infty - T(x,y,t) = (T_\infty - T_w)\left[1 - \frac{3}{2}\frac{y}{\delta_T} + \frac{1}{2}\left(\frac{y}{\delta_T}\right)^3\right] \tag{6}$$

is assumed. Here $\delta_T(x,t)$ is the thickness of the thermal boundary layer. The above profile is chosen so as to satisfy the conditions:

$$T(x,0,t,) = T_w,\quad T(x,\delta_T,t) = T_\infty,\quad \frac{\partial T}{\partial Y}(x,\delta_T,t) = 0,$$
$$\frac{\partial^2 T}{\partial y^2}(x,0,t) = 0 \tag{7}$$

Following the usual procedure of substituting Eqs. (4) and (6) into Eq. (1) and integrating Eq. (1) with y between the limits 0 and δ_T gives the equation

$$\delta_T\frac{\partial \delta_T}{\partial t} + u_\infty\,\delta_T\frac{\partial}{\partial x}\left[\delta\left\{\frac{2}{5}\left(\frac{\delta_T}{\delta}\right) - \frac{1}{35}\left(\frac{\delta_T}{\delta}\right)^4\right\}\right] = 4\alpha \tag{8}$$

The continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

has been employed in deriving Eq. (8).

Equation (8) admits similarity solutions. By introducing the variables,

$$\phi = \frac{\delta_T}{\delta} \quad\text{and}\quad \eta = \frac{u_\infty t}{x} \tag{10}$$

Eq. (8) can be rearranged as

$$\frac{\mathrm{d}\phi}{\mathrm{d}\eta} = \frac{\dfrac{C}{P_r} - \dfrac{1}{5}\phi^3\left(1 - \dfrac{1}{14}\phi^2\right)}{\phi\left[1 - \dfrac{4}{5}\phi\left(1 - \dfrac{\phi^2}{7}\right)\eta\right]} \tag{11}$$

where

$$C = \frac{4}{(4.64)^2} \quad\text{and}\quad P_r = \frac{\nu}{\alpha} = \text{Prandtl number} \tag{12}$$

The initial condition for Eq. (11) is obtained by noting that $\delta_T(x,0) = 0$ and $\eta(x,0) = 0.$, i.e.,

$$\phi(0) = 0 \tag{13}$$

Equation (11) is integrated numerically using a fourth order Runge-Kutta scheme. However, Eq. (11) has an infinite slope at the origin.

Since ϕ is small near the origin, all terms containing higher powers of ϕ can be neglected in Eq. (11) which can then be written as

$$\frac{\mathrm{d}\phi}{\mathrm{d}\eta} = \frac{C}{P_r\phi} \tag{14}$$

Integrating Eq. (14) using Eq. (13) gives the relation

$$\phi^2 = 2\frac{C}{P_r}\eta \tag{15}$$

This relation is used to prescribe the initial starting point for Eq. (11).

It is seen from Eq. (11) that the value ϕ_s which satisfies the relation

$$\frac{1}{5}\phi_s^3\left(1 - \frac{1}{14}\phi_s^2\right) = \frac{C}{P_r} \tag{16}$$

is a solution of Eq. (11). This does not satisfy the initial conditions of Eq. (13), but represents the steady-state solution of Eq. (11). This steady-state solution of Eq. (16) is obtained by an iterative scheme for $P_r = 0.72$ and is given by

$$\phi_s = 1.12348 \tag{17}$$

Equation (16) is identical to Eq. (5.9-32) in Whitaker,[4] where the incompressible steady-state thermal boundary layer problem for a flat plate is formulated and solved. The numerical value given by Eq. (17) is, however, different from that given in Whitaker because calculations in Eq. (17) were performed for $P_r = 0.72$.

If the steady-state solution of Eq. (11) is approached asymptotically $\mathrm{d}\phi/\mathrm{d}\eta$ should tend to zero as $\eta \to \infty$. But, numerical calculations of Eq. (11) indicate that $\mathrm{d}\phi/\mathrm{d}\eta$ decreases with η until $\eta \approx 1.03$ and then starts to increase. If the computations are continued beyond this value of η, the curve of ϕ vs η turns back on itself so that it is double valued for some values of η. This solution is physically unrealistic and the difficulty is caused by the mathematical nature of Eq. (11).

Goodman[5] resolved the above difficulty by assuming that the transient solution approaches the steady-state solution through a jump in the value of ϕ from $\phi = \phi_T$ to $\phi = \phi_s$ at some value of $\eta = \eta^*$. This jump was termed a heat wave, meaning that for $\eta > \eta^*$, the steady-state solution prevails and for $\eta < \eta^*$, the solution is transient. Physically, a step change in the fluid bulk temperature will set up a starting heat wave whose trajectory is given by $\eta = \eta^*$. After this wave has passed any particular station, however, the temperature steadies at that station.

To determine this value of η^*, the equation

$$\phi\left[1 - \frac{4}{5}\phi\left(1 - \frac{\phi^2}{7}\right)\eta^*\right]\frac{\mathrm{d}\phi}{\mathrm{d}\eta} = 0 \tag{18}$$

is integrated to give

$$\frac{1}{2}\phi_s^2 - \frac{4}{5}\left(\frac{\phi_s^3}{3} - \frac{\phi_s^5}{35}\right)\eta^* = \frac{1}{2}\phi_T^2 - \frac{4}{5}\left(\frac{\phi_T^3}{3} - \frac{\phi_T^5}{35}\right)\eta^* \tag{19}$$

on simplifying this expression, we have

$$\eta^* = \frac{5}{8}\frac{(\phi_s^2 - \phi_T^2)}{[(1/3)(\phi_s^3 - \phi_T^3) - (1/35)(\phi_s^5 - \phi_T^5)]} \tag{20}$$

However, the value of ϕ_T in Eq. (20) is not known. This is determined by an iterative procedure using Eqs. (11) and (20). A trial value of ϕ_T is chosen. Equation (11) is then integrated numerically from $\phi = 0$ to $\phi = \phi_T$ to determine η^*. This value

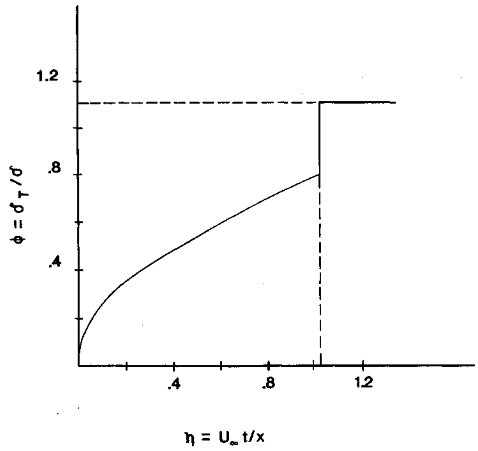

Fig. 3 Growth of thermal boundary layer with time.

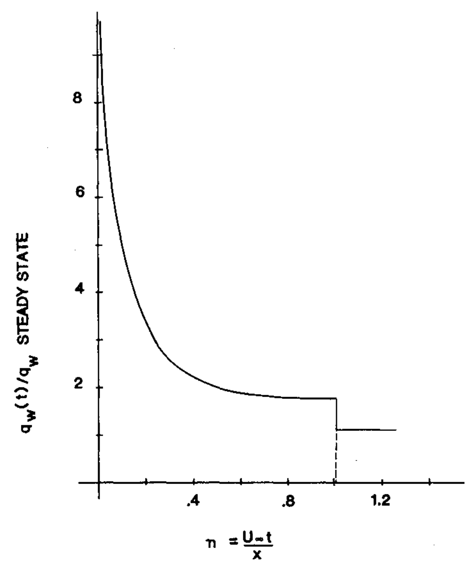

Fig. 4 Wall heat transfer as function of time.

is compared to the value calculated from Eq. (20). Further iterations on ϕ_T are used until these two values of η^* converge. The final calculated value of η^* is given by

$$\eta^* = \frac{u_\infty t}{x} = 1.035 \tag{21}$$

For values of $\eta > \eta^*$, we have the steady-state solution $\phi = \phi_s$ and for values of $\eta < \eta^*$, we have the transient solution of Eq. (11) for $0 \le \phi \le \phi_T$.

Results

The solution of Eq. (11) is displayed as a curve of (δ_T/δ) vs $(u_\infty t/x)$ in Fig. 3. The wave character at $\eta = \eta^*$ is represented by the jump of ϕ from ϕ_T to ϕ_s. Figure 3, in effect, represents the growth of the thermal boundary layer with time. The wall heat flux is given by

$$q_w(t) = -k\frac{\partial T}{\partial y}(x,0,t) = -\frac{3}{2}\frac{k(T_\infty - T_w)}{\phi\delta} \tag{22}$$

where k is the thermal conductivity of the fluid. Thus, the ratio of the instantaneous wall flux to the steady state wall flux is given by

$$\frac{q_w(t)}{q_w \text{ steady}} = \frac{\phi_s}{\phi} \tag{23}$$

This relationship is displayed in Fig. 4. Initially, the wall heat flux is very large compared to the steady-state value. The wall heat flux rapidly decreases with time, and at $\eta = \eta^*$, jumps to the steady-state value.

Discussion and Conclusions

The integral method used in this paper is an approximate method to solve the unsteady heat transfer problem. The parabolic partial differential equation (1) changes to a hyperbolic equation (11) because the integral method was used. A jump in the solution (shown in Figs. 3 and 4) occurs as a consequence of the change of the governing differential equation to hyperbolic type. We have not found an occurrence where this jump has been verified experimentally. The accuracy of the approximate solution cannot be determined presently due to the unavailability of more accurate solutions. A discussion by Goodman[5] on the accuracy of integral methods indicates that accuracies of ± 15 percent can be expected.

We hope that this analysis will be helpful in making some engineering calculations for the laser flow loop. It is recognized that the thermal inertias of solid surfaces are large compared to the gas flow thermal inertia in a laser flow loop.[6] The assumption of constant wall temperature is, therefore, justified. The application of boundary layer suction just before the flow enters the cavity (Fig. 1) is a common practice.[7] A new boundary layer starts developing on the cavity walls. This boundary layer is laminar in many designs. When the cavity flow is subjected to a step change in temperature, the growth of the thermal boundary layer and wall heat transfer can be estimated using the present analysis. The heat exchanger, which is subjected to the unsteady temperature field (shown in Fig. 2), is sometimes crudely modeled as a flat plate. Also, the analysis can be used to estimate the boundary layer growth and wall heat transfer for a single pulse.

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